

# The Dynamics of Water Ultrafiltration Flow within Framework of Hydrodynamical Approach

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## Abstract

The simplest model of adsorbing of fouling for ultrafiltration is considered the simplest model of solutes adsorption and fouling for ultrafiltration was considered. Within framework of hydro dynamical approach the membrane resistance was obtained. The membrane resistance was obtained within a framework of hydrodynamics. It was found that membrane resistance is non-exponential function on membrane operation time. The membrane resistance was found to be a non-exponential function on operation time. We found that for a short time of membrane operation the flux drops as a linear function. We found the flux drops as a linear function during a short time of membrane operation. For a long time  $t$  of operation the flux decline is proportional to  $t^2$ .

**Keywords:** Ultrafiltration; Membrane resistance

## Introduction

Ultrafiltration (UF) is a variety of membrane filtration in which forces like pressure or concentration gradients lead to a separation through a semi permeable membrane. Ultrafiltration (UF) is a type of membrane filtration in which pressure or concentration gradients separate through a semi-permeable membrane. Suspended solids and solutes of high molecular weight are retained in the so-called retentate, while water and low molecular weight solutes pass through the membrane in the permeate. During the process suspended solids of heavy molecular weight are retained while water and lighter molecular weight solutes pass through the membrane. This separation process is used in industry and research for purifying and concentrating macromolecular  $10^3 \div 10^6$  Da (Dalton) ( $1\text{Da}=1.660538921(73) \cdot 10^{-27}$  kg is the unified atomic mass unit) solutions, especially protein solutions. Ultrafiltration is not fundamentally different from microfiltration. Both of these separate based on size exclusion or particle capture. Both processes segregate based on molecular size of particles. It is fundamentally different from membrane gas separation, which separate based on different amounts of absorption and different rates of diffusion. The process is different from membrane gas separations which segregate based on levels of absorption and diffusion. Ultrafiltration membranes are defined by the molecular weight cut-off (MWCO) of the membrane used. Ultrafiltration is applied in cross-flow or dead-end mode. The size of membrane interstices lies within range from 5 nm to 100 nm. The interstices membrane is within 5 nm to 100 nm.

At present, Ultrafiltration begins to play a significant role for drinking water production. In spite of the ultrafiltration grew fast during few last years, the membrane fouling problem is critical issue. Not sure what this means? The basic operating principle of ultrafiltration uses a pressure induced separation of solutes from water through a semi permeable membrane. The relationship between the applied pressure on the solution of water to be separated and the flux through the membrane is most commonly described by the Darcy Equation 1:

$$J = \frac{\Delta P}{\mu R_t} \dots\dots\dots(1)$$

where  $J=v$  is the flux (flow rate per membrane area),  $\Delta P$  is the transmembrane pressure (pressure difference between feed and permeate stream),  $\mu$  is the water viscosity,  $R_t$  is the total resistance (sum of membrane and fouling resistance). During the operating time of the membrane specific interactions between the membrane and the components in the raw water arise and cause a rapid and often irreversible fouling of membrane pores. A rapid and often irreversible fouling of the membrane pore is caused during ultrafiltration. As a consequence, the fouling resistance  $R$  grows and a rapid decline of flux  $J$  through the membrane takes place.

At present the processes of ultrafiltration are given detailed investigations [1-5]. In papers of Ho C-C et al. [6] and Andrianov AP et al. [7] the empiric formulae, which describe evolution of water flux  $J$  passing through the membrane, were obtained by using experimental data.

In our paper we apply the theoretical approach and consider the simplest model of UF membrane in order to obtain evolution of flux  $J$  with operating time  $t$ .

## Basic Equations and its Solutions

We investigate the dynamics of water Ultrafiltration flux within framework of hydrodynamical approach. We assume that a membrane with thickness  $l$  and area  $S$  has the  $n$  pores. Each interstice is a straight channel of cylindrical form with the diameter  $d$  and the length  $l$ . The total cross section area of all pores is  $S_n = \frac{n\pi d^2}{4}$ . The continuity equation gives Equation 2

$$uS_n = vS_n \text{ or } u = \xi \left( \frac{d_0}{d} \right)^2 v \quad \dots\dots\dots(2)$$

Where  $u$  is an average bulk velocity of water in the channel (within membrane) and is a velocity of water outside the membrane. We introduce the ratio between the area of membrane  $S$  and total cross section area of all pores for a clean membrane:  $\xi = \frac{S}{S_{n0}}$ , where  $S_{n0} = \frac{\pi d_0^2}{4}$  and  $d_0$  is an initial diameter of the channel of a clean membrane.

We will describe the percolation of water through the membrane by the Bernoulli equation:

$$\Delta P = \alpha_1 \frac{\rho v^2}{2} + \lambda \frac{l}{d} \frac{\rho v^2}{2} + \alpha_2 \frac{\rho v^2}{2} \quad \dots\dots\dots(3)$$

Here  $P$  is a mass density of water,  $\Delta P$  is the transmembrane pressure (pressure difference between feed and permeate stream),  $\alpha_1$  is the resistance coefficient at the entrance of the interstice and  $\alpha_2$  is the resistance coefficient at the outlet of the interstice;  $\lambda$  describes the resistance along the whole length. The Reynolds number for water flow in channel is

$$Re = \frac{ud}{\nu} = \frac{vd_0^2}{\nu d} \quad \dots\dots\dots(4)$$

where  $\nu$  is kinetic viscosity of water. For laminar flow in a channel the Reynolds number is smaller than 2300 and the coefficient of resistance  $\lambda$  is described by the Poiseuille formula. In opposite case, when the flow in the channel is a vortex, the coefficient of the resistance  $\lambda$  is described by the Altshul formula. So we can write:

$$\lambda = \begin{cases} \frac{64}{Re} & Re < 2300, \\ 0.11 \left( \frac{\Delta}{d} + \frac{68}{Re} \right)^{0.25} & Re > 2300 \end{cases} \quad \dots\dots\dots(5)$$

Here  $\Delta$  is an apparent roughness of the internal surface of the pores.

The diameter of membrane pore changes during the filtration process as a result of solutes adsorption at the internal surface of the pores. If the  $k$  is a coefficient of filtration (the ratio of adsorbed matter to solvent matter), we can suggest, that the mass adsorbed absorbed, please use spell check within a pore is proportional to the amount of permeate volume of purified water, that is

$$\dot{m} = kc_f Sv, \quad \dots\dots\dots(6)$$

where  $c_f$  is a concentration of solutes,  $J = V = Sv$  is flux through the membrane. The point over symbols denotes derivation over time  $t$ . Following the authors of the paper [8] again state what's in Katsoufidou's paper, e.g [8] found that, we assume that fouling mass adsorbs uniformly forming a cylinder:

$$m = \frac{\pi}{4} (d_0^2 - d^2) l \rho_f n, \quad \dots\dots\dots(7)$$

where  $\rho_f$  is the average mass density of the fouling adsorbed. Using (7) we can rewrite the equation (6) for  $d$  as

$$\dot{d} = \frac{2}{\pi} \frac{kc_f Sv}{l \rho_f dn} = -k \frac{c_f}{\rho_f} \frac{\xi d_0^2}{2l} \frac{v}{d} \quad \dots\dots\dots(8)$$

This differential equation is added to the initial condition  $d(0) = d_0$ . The equations (2-5), (8) describe process of raw water percolation through the Ultrafiltration membrane.

We assume that in the pore the Reynolds number  $Re < 1$  and the water flow is laminar. The coefficient  $\lambda$  is determined by the Poiseuille formula and the resistance along the whole length  $l$  is larger than  $\alpha_1 + \alpha_2$ . Neglecting the terms with  $\alpha_1$  and  $\alpha_2$  we find the velocity of the flow from the equation (3):

$$v = \frac{\Delta P d^4 \pi}{32 \mu \xi d_0^2 l} \quad \dots\dots\dots(9)$$

Here  $\mu = \rho \nu$  is a dynamic solvent viscosity. Substituting (9) in the equation (8) and then integrating it the evolution of the diameter  $d$  with time  $t$  is found:

$$d(t) = \frac{d_0}{\sqrt{1 + k \frac{c_f}{\rho_f} \frac{\Delta P d_0^2}{32 \mu l^2} t}} \quad \dots\dots\dots(10)$$

Evolution of the flux per unit area of membrane is

$$v(t) = \frac{\Delta P d_0^2}{32 \mu \xi l \left( 1 + k \frac{c_f}{\rho_f} \frac{\Delta P d_0^2}{32 \mu l^2} t \right)^2} \quad \dots\dots\dots(11)$$

Comparing the equation (1) with the equation (11) we find the total resistance of the membrane:

$$R_t = \frac{32 \xi l}{d_0^2} \left( 1 + k \frac{c_f}{\rho_f} \frac{\Delta P d_0^2}{32 \mu l^2} t \right)^2 \quad \dots\dots\dots(12)$$

In this case the Reynolds number is

$$Re = \frac{\Delta P d_0^2}{32 \mu \nu l \left( 1 + k \frac{c_f}{\rho_f} \frac{\Delta P d_0^2}{32 \mu l^2} t \right)^{\frac{3}{2}}} \leq \frac{\Delta P d_0^3}{32 \mu \nu l} \quad \dots\dots\dots(13)$$

For real conditions, when a filtration pressure is  $5 \cdot 10^5$  Pa, thickness of membrane is  $l = 0.01$  mm and the diameter of pores is  $d_0 = 100$  nm, the dimensionless parameter is equal to

$$\frac{\Delta P d_0^3}{32 \mu \nu l} = 0.0015 \quad \dots\dots\dots(14)$$

So, the Reynolds number  $Re \ll 1$  in the pore and the parameter  $\lambda l / d \gg 1$ . This means that our original suggestion that water flow is a laminar is realized.

We found that the resistance  $R_f(t)$  and the flux  $v(t)$  have non-exponential low. For a small time of operation, when inequality

$$t \ll \frac{\rho_f}{c_f} \frac{32 \mu l^2}{k \Delta P d_0^2} \quad \dots\dots\dots(15)$$

Holds, evolution of the total membrane resistance has linear law

$$R_t = \frac{32 \xi l}{d_0^2} \left( 1 + 2k \frac{c_f}{\rho_f} \frac{\Delta P d_0^2}{32 \mu l^2} t \right) \quad \dots\dots\dots(16)$$

$R_f = \frac{32 \xi l}{d_0^2}$  is the resistance of a clean membrane. The flux per unit area of membrane drops as a linear function:

$$v(t) \approx \frac{\Delta P d_0^2}{32 \mu \xi l} \left( 1 - 2k \frac{\Delta P d_0^2}{32 \mu l^2} t \right) \quad \dots\dots (17)$$

Here  $v = \frac{\Delta P d_0^2}{32 \mu \xi l}$  is an initial flux. For a long time of operation, when inequality

$$t \gg \frac{\rho_f}{c_f} \frac{32 \mu l^2}{k \Delta P d_0^2} \quad \dots\dots (18)$$

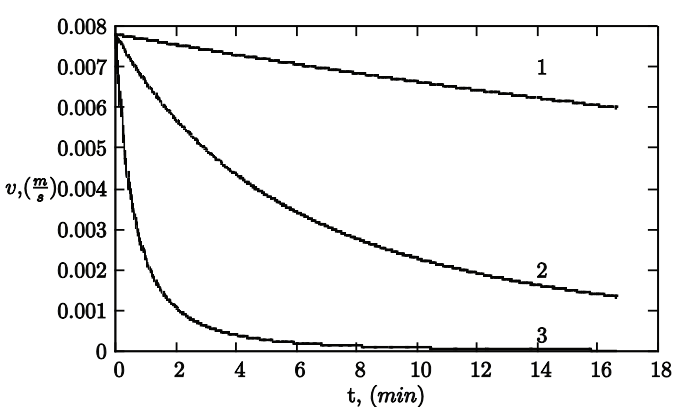
Holds, the total resistance of the membrane increases as  $t^2$  and equals

$$R_f = \frac{\xi}{32 l^2} \left( k \frac{\rho_f}{c_f} \frac{\Delta P d_0^2}{\mu} \right)^2 t^2 \quad \dots\dots(19)$$

Water flux per through the membrane per unit area of membrane is

$$v(t) = \frac{32 \mu l^2}{\xi \left( k \frac{c_f}{\rho_f} \right) \Delta P d_0^2} \quad \dots\dots(20)$$

We can see that evolution of water flux is proportional to  $t^2$ . In figure 1 the results of calculations are presented for following parameters:  $d_0 = 100 \text{ nm}$ ,  $\Delta P = 10^5 \text{ Pa}$ ,  $k = 0.9$ ,  $l = 0.01 \text{ mm}$ . Curve 1 shows flux evolution for  $\frac{c_f}{\rho_f} = 10^{-5}$ , curve 2 shows flux evolution for  $\frac{c_f}{\rho_f} = 10^6$  and curve 3 shows flux evolution for  $\frac{c_f}{\rho_f} = 10^7$ . One can see that for the curve 1 the inequality (15) holds, and flux drops as a linear function (17), while for the curve 3 the inequality (18) takes place, and flux drops as  $t^2$  according to formula (20).



**Figure 1:** The flux of water per unit area  $v(t)$  for following Parameters:  $d_0 = 100 \text{ nm}$ ,  $\Delta P = 10^5 \text{ Pa}$ ,  $k = 0.9$ ,  $l = 0.01 \text{ mm}$ . Curve 1 for  $\frac{c_f}{\rho_f} = 10^{-5}$ , curve 2 for  $\frac{c_f}{\rho_f} = 10^6$ , curve 3 for  $\frac{c_f}{\rho_f} = 10^7$ .

### Conclusions

We considered evolution of water flux passing through the Ultrafiltration membrane. We applied the hydrodynamical approach and found that membrane resistance is a non-exponential function on membrane operation time. We found that for a short time of membrane operation the flux drops as a linear function. For a long time  $t$  of operation the flux decline is proportional to  $t^2$ .

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